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Beam stability on an elastic foundation subjected to distributed follower force †

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Abstract

Problems related to the stability and behavior of a cantilevered beam with a tip mass on an elastic foundation and subjected to a distributed follower force are addressed. The stability of a beam partially attached to an elastic foundation is also considered. The dynamic stability of a beam subjected to a distributed follower force is formulated by using finite element method to get a general eigenvalue problem. The influence of the modulus on the elastic foundation and the ratio of the cantilevered beam's mass to the tip mass on the critical flutter are investigated. Finally, the stability of the cantilevered beam is found to depend on both the modulus of the elastic foundation and the ratio of the cantilevered beam's mass to the tip mass.

Keywords: Stability; Distributed follower force; Finite element method; Elastic foundation; Flutter

1. Introduction

An elastic stability problem of non-conservative system is widely used for civil engineering as well as aeronautical and space science, mechanical engineering, architectural engineering, so it attracts greater interest in this field. In 1952, Beck started to work on the stability of columns for concentrated follower force effects along the tangential direction at the tip of a cantilevered column as in Fig. 1 [1]. This research introduced a consideration of the mass on a cantilevered column at the free end [2] as in Fig. 2, and the parametric studies were implemented. Also, a follower force is not only a concentrated one, but also an applied force that is equally distributed to the limit length of the column. Leipholz studied a distributed follower force (Fig. 3) for the first time [3]. Many researches on concentrated follower force and distributed follower force have been extended, and they are represented by Langthjem and Sugiyama's paper [4].

However, among these studies, there exist some inadequately investigated problems. The representative example in these studies is, as first studied in 1972 by Smith and Herrmann, the effect of an elastic foundation parameter influence on the stability of a cantilevered beam subjected to a concentrated follower force on an elastic foundation [5] as in Fig. 4.

They found that the critical follower force of the cantilevered beam does not depend on the elastic foundation modulus, and this unexpected result became the beginning of elastic foundation beam studies which have been pursued till now.

In 1975, Anderson investigated the influence of rotatory inertia, tip mass and internal damping on the stability of an elastic foundation beam subjected to a concentrated follower force [6]. Yoon and Kim studied the influence of inertial moment of tip mass on the stability of Beck's column [7]. Moreover, Lee and his co-researchers studied the stability of elasticity supported Timoshenko's beam [8]. Recently, Maurizi and Bambill verified again the stability of beams placed

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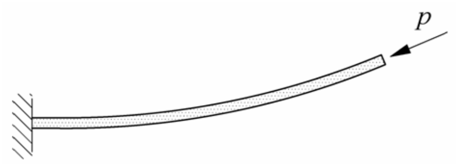


Fig. 1. Cantilevered column subjected to a concentrated follower force (Beck's column).

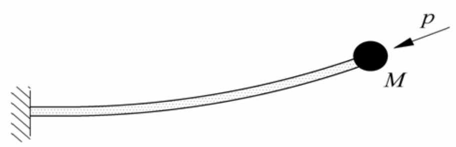


Fig. 2. Cantilevered column subjected to a concentrated follower force with a tip mass (Pfluger's column).

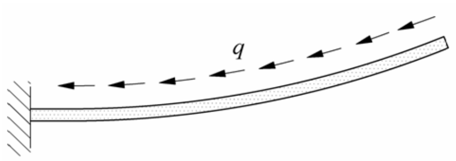


Fig. 3. Cantilevered column subjected to a distributed follower force (Leipholz's column).

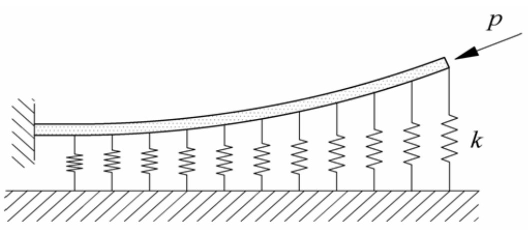


Fig. 4. Cantilevered beam subjected to a concentrated follower force on an elastic foundation (Smith & Herrmann's beam).

on an elastic foundation [9]. All these studies are, however, limited to in the case that the concentrated follower force works.

Therefore, this study focused on when a distributed follower force acts, an elastic foundation exists all over beams, or an elastic foundation exists partially, an influence of elastic foundation to the stability of beams with a tip mass.

2. Theory analysis

2.1 Structural model

Fig. 5 and Fig. 6 depict the structural model on an

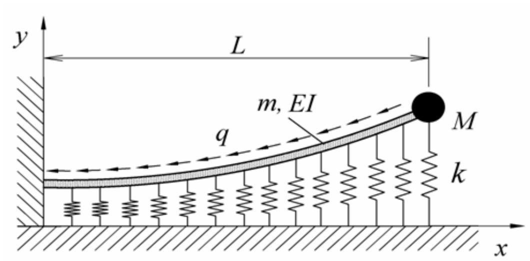


Fig. 5. Cantilevered beam on elastic foundation subjected to distributed follower force with a tip mass.

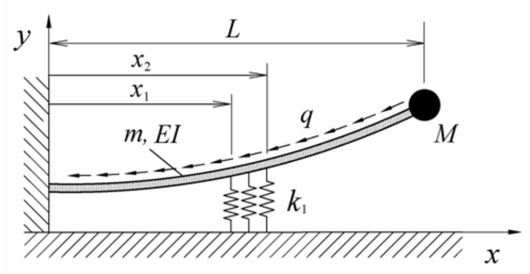


Fig. 6. Cantilevered beam on partial elastic foundation subjected to distributed follower force with a tip mass.

elastic foundation exerted by a distributed follower force.

Fig. 5 reveals a cantilevered beam on elastic foundation subjected to distributed follower force with a tip mass, and Fig. 6 shows a cantilevered beam on partial elastic foundation subjected to distributed follower force with a tip mass.

The beam is assumed to be an elastic straight beam of total length L, mass per unit length m, bending stiffness EI, tip mass M. And the distributed follower force q is assumed to be linearly distributed along the x-axis as follows.

The inertial moment of tip mass and internal damping will not be considered in the present paper.

Here a computational approach by applying the finite element method (for example, see Ref. [10]) is adopted to investigate the eigenfrequency and the critical distributed follower force.

2.2 Equation of motion and boundary condition

The extended Hamilton's principle for a nonconservative system under consideration can be written in the form

$$\delta\int_{t_1}^{t_2}(T+W_c-U)\,dt+\int_{t_1}^{t_2}(\delta W_{nc})\,dt=0\,, \qquad (1)$$

where T is the total kinetic energy of the system, W_c the work done by the conservative component of the distributed follower force, U is potential energy of the beam due to bending, and the virtual work done by the non-conservative component of the distributed follower force.

The energy and work components are given as follows (for example, see Ref. [11], [12]):

$$T = \frac{1}{2}\int_0^L m\left(\frac{\partial y}{\partial t}\right)^2 dx + \frac{M}{2}\left(\frac{\partial y}{\partial t}\right)^2,$$

$$W_c = \frac{1}{2}\int_0^L q(L-x)\left(\frac{\partial y}{\partial x}\right)^2 dx,$$

$$U = \frac{1}{2}\int_0^L EI\left(\frac{\partial^2 y}{\partial x^2}\right)^2 dx + \frac{1}{2}\int_0^L k\,y^2 dx + \frac{1}{2}\int_{x_1}^{x_2} k_1 y^2 dx,$$

$$\delta W_{nc} = -\int_0^L q\left(\frac{\partial y}{\partial x}\right)\delta y dx. \quad (2)$$

For simplicity the following dimensionless quantities are introduced:

$$\xi = \frac{x}{L}, \quad \eta = \frac{y}{L}, \quad \tau = \frac{t}{L^2}\sqrt{\frac{EI}{m}}, \quad \alpha = \frac{x_2 - x_1}{L},$$

$$\mu = \frac{M}{mL}, \quad \kappa = \frac{kL^4}{EI}, \quad \kappa_1 = \frac{k_1 L^3}{EI}, \quad \rho = \frac{qL^3}{EI}, \quad (3)$$

where ρ is the non-dimensional distributed follower force parameter, κ is the non-dimensional spring constant of the elastic foundation. Further, κ_1 is the non-dimensional spring constant of the beam partially attached to an elastic foundation.

Substitution of Eqs. (2) and (3) into equation (1) yields, after rearrangements and transformations,

$$\int_{\tau_1}^{\tau_2}\int_0^1 [\eta_\tau\delta\eta_\tau + \rho(1-\xi)\eta_\xi\delta\eta_\xi - \eta_{\xi\xi}\delta\eta_{\xi\xi} - \rho\eta_\xi\delta\eta - \kappa\eta\delta\eta]d\xi d\tau - \int_{\tau_1}^{\tau_2}\left[(\kappa_1\eta\delta\eta)_{\xi=\alpha} + (\mu\eta_\tau\delta\eta_\tau)_{\xi=1}\right]d\tau = 0. \quad (4)$$

In order to obtain a characteristic equation of small motion of the beam, the beam is divided into *N* equal elements to be compatible with the finite element method.

Then equation (4) can be written as

$$\int_{\tau_1}^{\tau_2}\left[\sum_{i=1}^{N}\int_{\frac{1}{N}(i-1)}^{\frac{1}{N}i}\{\eta_\tau\delta\eta_\tau + \rho(1-\xi)\eta_\xi\delta\eta_\xi - \eta_{\xi\xi}\delta\eta_{\xi\xi} - \rho\eta_\xi\delta\eta - \kappa\eta\delta\eta\}d\xi - \left\{(\kappa_1\eta\delta\eta)_{\xi=\alpha} + (\mu\eta_\tau\delta\eta_\tau)_{\xi=1}\right\}\right]d\tau = 0. \quad (5)$$

Substitution of local coordinates $(\zeta = N\xi - i + 1)$ into Eq. (5) yields the following discretized equation:

$$\int_{\tau_1}^{\tau_2}\left[\sum_{i=1}^{N}\{\eta_\tau^{(i)}\delta\eta_\tau^{(i)} + \rho N(N-i+1-\xi)\eta_\xi^{(i)}\delta\eta_\xi^{(i)} - N^4\eta_{\xi\xi}^{(i)}\delta\eta_{\xi\xi}^{(i)} - \rho N\eta_\xi^{(i)}\delta\eta^{(i)} - \kappa\eta^{(i)}\delta\eta^{(i)}\}d\xi - \left\{(\kappa_1\eta^{(i)}\delta\eta^{(i)})_{\xi=\alpha} + (\mu\eta^{(N)}\delta\eta^{(N)})_{\xi=1}\right\}\right]d\tau = 0. \quad (6)$$

The dimensionless displacement η can be assumed to take the form

$$\eta^{(i)}(\zeta,\tau) = \mathbf{e}^{(i)}(\zeta)\cdot\mathbf{v}^{(i)}(\tau), \quad (7)$$

where $\mathbf{e}^{(i)}(\zeta)$ is a vector of cubic shape functions, and $\mathbf{v}^{(i)}(\tau)$ is a vector of nodal displacements. By substitution of Eq. (7) into (6), finally the characteristic equation is obtained in matrix form

$$\mathbf{M}\mathbf{v}_{\tau\tau} + \mathbf{K}\mathbf{v} = \mathbf{0}, \quad (8)$$

where $\mathbf{M}$ is a symmetric mass matrix, $\mathbf{K}$ is a non-symmetric stiffness matrix.

The displacement field varies with time according to an exponential law:

$$\mathbf{v}(\tau) = \mathbf{X}exp(\lambda\tau). \quad (9)$$

Finally, the global characteristic equation can be obtained in the form

$$\left|\mathbf{M}^{-1}\mathbf{K} + \lambda\mathbf{E}\right| = \mathbf{0}, \quad (10)$$

where $\mathbf{E}$ is the unit matrix.

The stability of the system under consideration is determined by the sign of real part σ of the complex eigenvalue $\lambda = \sigma \pm i\omega$ $(i=\sqrt{-1})$. If $\sigma < 0$,the system is stable; if $\sigma > 0$ and $\omega = 0$, the system is statically unstable, i.e., divergence type instability; if

$\sigma > 0$ and $\omega \neq 0$, the system is dynamically unstable, i.e., flutter type instability; if $\sigma = 0$, the critical distributed follower force (ρ_{cr}) arises.

3. Numerical results

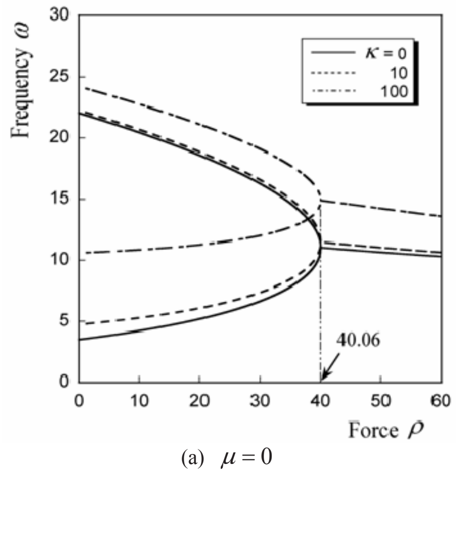
In this paper, it is assumed that the tip mass and the elastic foundation are coupled.

Fig. 7 shows the first and second eigenfrequencies for the change of the non-dimensional distributed follower force, with the non-dimensional spring constants of the elastic foundation, κ=0, 10 and 100, for the totally attached to an elastic foundation beam.

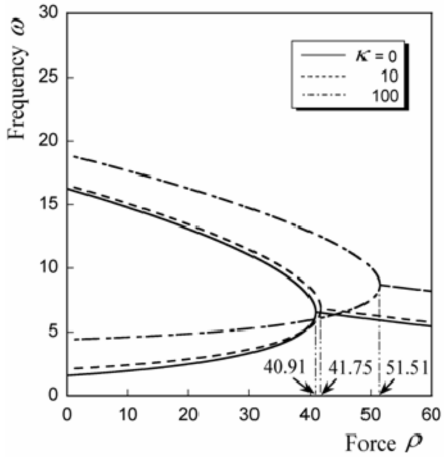
In Fig. 7(a), in case of μ = 0, as the non-dimensional spring constant κ increases, the first and second eigenfrequencies also increase. The first and second eigenfrequencies coincide with each other and thus flutter instability occurs. The value of the non-dimensional critical distributed follower force ρ_{cr} is 40.06 for all the values of κ.

This result is the same as the result of the bibliography [5]; when the concentrated follower force is in effect, the non-dimensional critical concentrated follower force is independent of the variation on the value of κ.

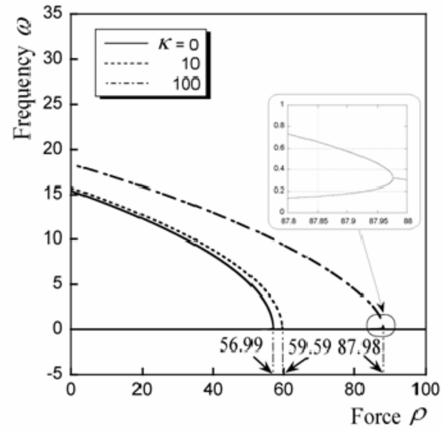
Fig. 7(b) and (c) show the behavior of the first and second eigenfrequencies for μ = 1 and 10^6 respectively. As the value of κ increases, the value of the non-dimensional critical distributed follower force which occurs by correspondence of the first and second eigenfrequencies and its eigenfrequencies also increases. It is also found that the first and second eigenfrequencies decrease, as the value of μ increases.



(a) $\mu = 0$



(b) $\mu = 1$



(C) $\mu = 10^6$

Fig. 7. The first and second eigenfrequencies for the change of the non-dimensional distributed follower force.

Fig. 8 shows the non-dimensional critical distributed follower force ρ_{cr} as a function of tip mass parameter μ for the elastic foundation parameter κ=1, 10, 100, respectively.

The figure represents that the non-dimensional critical distributed follower force decreases until the value of the tip mass parameter μ reaches 0.1, and the critical distributed follower force increases in the range that μ is larger than 0.2.

Moreover, the elastic foundation has a destabilizing effect for $0 < \mu \leq 0.1$; the stabilizing effect of the elastic foundation strongly appears with increasing the value of tip mass parameter μ in the range that μ is larger than 0.2.

Fig. 9 shows the non-dimensional critical eigenfrequency ω_{cr} as a function of tip mass parameter μ

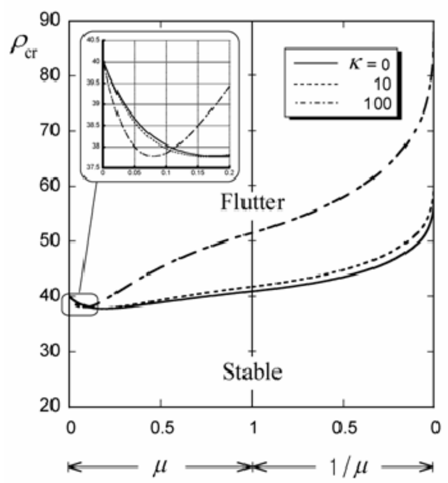


Fig. 8. Non-dimensional critical distributed follower force as a function of tip mass parameter for the elastic foundation parameter κ =1, 10, 100.

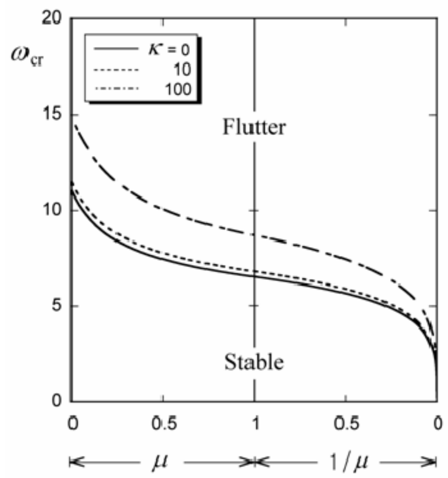
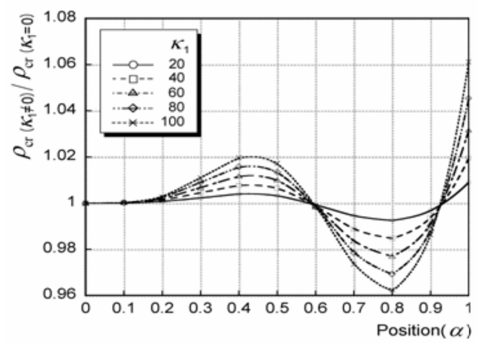


Fig. 9. Non-dimensional critical eigenfrequency as a function of tip mass parameter for the elastic foundation parameter κ =1, 10, 100.

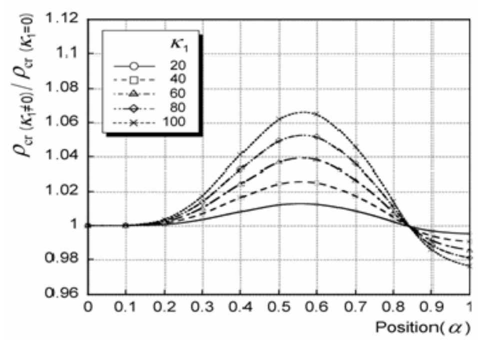
for the elastic foundation parameter κ =1, 10, 100, respectively.

From this figure, in all the range of μ , the value of the critical eigenfrequency is large as the value of κ is large. Also, the increase of μ clearly reveals a decrease of the value of the critical eigenfrequency.

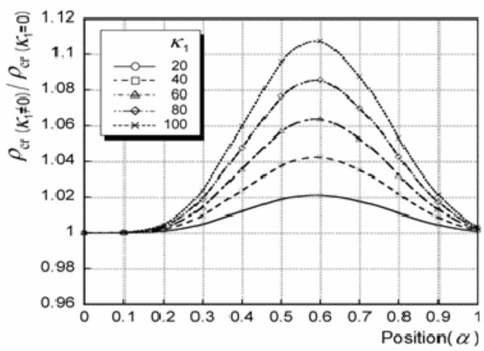
From Fig. 10 it is also seen that when a beam is partially attached to an elastic foundation, the variation of location α of an elastic foundation makes the value of the follower force to be determined for the non-dimensional spring constants κ_1 = 0, 20, 40, 60, 80, 100.



(a) $\mu = 0$



(b) $\mu = 1$



(c) $\mu = 100$

Fig. 10. Non-dimensional distributed follower force depending on the position of elastic foundations for k_1 =20, 40, 60, 80, 100.

In Fig. 10(a), the distributed follower force is applied, and tip mass parameter μ ignored. As the value of κ_1 increases, the critical distributed follower force gradually increases until the value of α reaches 0.6; the critical distributed follower force decreases when α is between 0.6 and 0.9, while it increases when the value of α is bigger than 0.9.

In Fig. 10(b) is shown that the distributed follower force is given and tip mass parameter μ is 1. From the figure, the critical distributed follower force increases as the value of κ_1 increases until the value of α reaches 0.8, and the critical distributed follower force becomes the largest value when α is between 0.5 and 0.6. If α is bigger than 0.8, the critical distributed follower force decreases as κ_1 increases.

In Fig. 10(c), when the distributed follower force is applied and tip mass parameter μ is 100, the critical distributed follower force increases as κ_1 increases in all sections. The largest critical distributed follower force appears when α is 0.6.

4. Conclusions

A study devoted to stability behavior of a beam totally or partially attached to an elastic foundation, to which a distributed follower force applies, was implemented.

The implications of a number of effects that have a profound role on their instability have been put into evidence. That is, as an elastic foundation parameter gets bigger, the flutter instability is influenced by the tip mass, and the distributed follower force. If the tip mass increases, the value of the critical eigenfrequency decreases; and also if an elastic foundation increases, the value of the critical eigenfrequency increases even if the tip mass is not considered.

In addition, if there is an elastic foundation in partial part of a beam, both a destabilizing effect and stabilizing one become bigger as the non-dimensional spring constant increases.

It is expected that the results and conclusions reported herein will be beneficial to those engaged in the study, design and implementation of a beam on a foundation subjected to a follower force and a tip mass.

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